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Chua Chaotic System Parameters Estimation using PSO Algorithm to increase its Dynamics

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Abstract

In this article, we estimate the parameters of the Chua system using PSO optimization algorithm by exploiting the property of chaotic synchronization; in order to increase the chaotic dynamics and this is done by the modifications of the parameters of a nonlinear dynamical system to obtain optimal parameter values that result in the most chaotic system. Parameter estimation is formulated as a multidimensional optimization problem that aims to minimize the synchronization error between two chaotic systems. To verify the accuracy and robustness of the proposed algorithm in parameter estimation, a Chua system is simulated and comparison experiments are performed. Through the results obtained, the efficiency of the algorithm was efficient in the estimation of the parameters, where we obtained a more chaotic system at the estimated values.

Keywords: Chaos; Chua system; Estimation; PSO; Synchronization

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1. Introduction

The term "chaos" defines a particular state of a system whose behavior never repeats itself. There are several possible definitions of chaos [1], [2]. Due to the properties of chaotic systems, such as the very sensitivity of nonlinear systems to initial conditions and their parameters, a small change in the parameters of chaotic systems can lead to another system [3], [4]. Chaotic systems have been applied in various fields such as communication systems [4], assurance of a communication system increases with the high chaotic system randomness caused through the increase of the chaotic dynamics.

Parameter estimation for chaotic systems has become an important problem in last decades with the aim of increasing the dynamics of chaos. There are many parameter estimation methods, some researches have also estimated chaotic system parameters using synchronization based methods like feedback-based synchronization method and adaptive synchronization [6], [7].

Parameter estimation can be formulated as a multi-dimensional. Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) algorithm are used to estimate the parameters of the chaotic Lorenz system [8], [9], [10], and Differential Evolution (DE) [11], and the Gravity Search Algorithm (GSA) [12] are adopted to estimate the parameters of the chaotic Lorenz system. PSO has been applied to estimate chaotic Lorenz systems parameters in a wide research range [13]. The parameter estimation method is based on the hybridization between the synchronization error and optimization algorithms, because the synchronization error is very sensitive to the chaotic system parameters. The smaller the synchronization error, the better is the parameters estimation. For this purpose, we minimize the synchronization error considered as an optimization problem.

In this article, we will apply the PSO algorithm to estimate parameters of the chaotic Chua system to minimize the synchronization error. This paper is organized as follows: In the first part, we address a brief introduction to the proposed PSO algorithm. In the second part, we present the problem of the estimation of the parameters by giving the mathematical formula. In the third part, we estimate the parameters of the Chua circuit, and in the fourth part, we present and discuss the simulation results, and finally we conclude this work by some deductions.

2. PSO Algorithm

PSO is a technique proposed by Kenndy and Aberhard in 1995 [14], which is inspired by the social behavior of animals moving in swarms. Authors in [15] have also given recent insight about PSO. This algorithm was found to be effective in solving the optimization problems characterized by their non-linearity and non-differentiation, multi-optima and high dimensionality through adaptation.

The PSO optimizes an objective function by performing a search based on the population P. The population is composed of potential solutions, called p-particles, initialized randomly and flying freely in the multidimensional search space E. During the flight, the particles change their own positions p(t) and speed v(t) according to their own experiments where the cost of the function is optimal f(t). Eventually, all the particles will gather around the global optimal solution$\vec{G}_{best}$. The motion of the particles between iteration t and t+1 is calculated using equations (1) and (2).

$$v_i(t+1) = wv_i(t) + c_1 r_1\big(p_i(t) - x_i(t)\big) + c_2 r_2\big(p_i(t) - x_i(t)\big) \quad (1)$$

$$x_i(t+1) = x_i(t)+v_i(t+1) \quad (2)$$

Where: w:InertiaCoefficient

$c_1$etc_2 : correlation coefficients

$r_1$et r_2 : uniformly distributed random numbers in [0,1]

3. Parametric Estimation Problem

Parameters estimation can be formulated as a nonlinear multidimensional optimization problem to minimize the objective function J for the parameter decision vector θ. The general scheme is shown in Fig.1.Consider the dynamical system of dimension n and parameters m:

$$\dot{x} = f(x, x_0, \theta) \quad (3)$$

Where:

$x \in R^n$isthe state vector with dimension n, $x_0$initial state and$\theta \in R^m$is the m dimension vector of system parameters. To estimate the following system parameters:

$$\dot{\hat{x}} = f(\hat{x}, x_0, \hat{\theta}) \quad (4)$$

Where :

$\hat{x} \in R^n$: the state estimation vector with dimension n

x_0 : the initial state and$\hat{\theta} \in R^m$is the system parameters estimation vector with dimension m.

The parameter estimation method is based on the synchronization error between the transmitter and the receiver because the synchronization error is very sensitive to the parameters of the chaotic system. For this reason, we minimize the synchronization error, which we consider as the objective function J representing the root mean square of the synchronization error.

$$J = \frac{1}{M}\sum_{K=1}^{M} \|x_k - \hat{x}_k\|^2 \quad (5)$$

M:the number of states

x_k: states of the master system

$\hat{x}_k$: states of the slave system

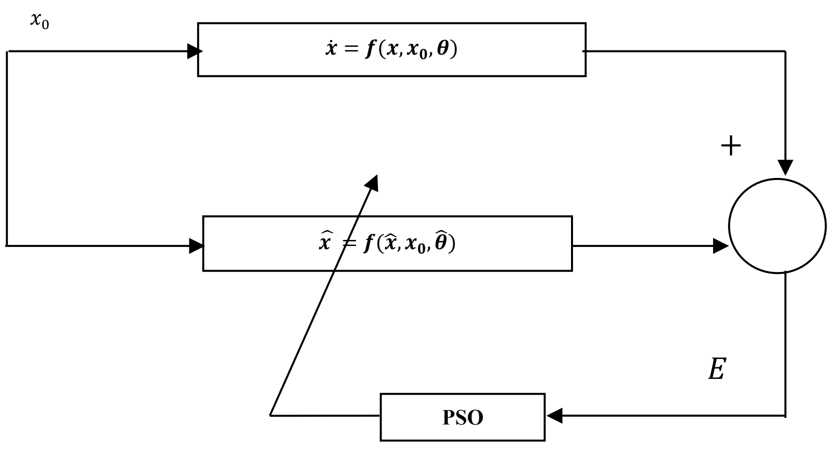

Figure 1. General Scheme of parameters estimation in chaotic systems

4. Application example

In this section, we apply PSO Algorithm in order to estimate the Chua Chaotic system parameters.

4.1 Chua System

Chua circuit is a simple electrical circuit that shows the classical behavior of Chaos theory [16], [17] and [18]. It has been presented byLeon O. Chua, an invited Professor at Waseda University in Japan. The proposed model is shown in figure Fig.2:

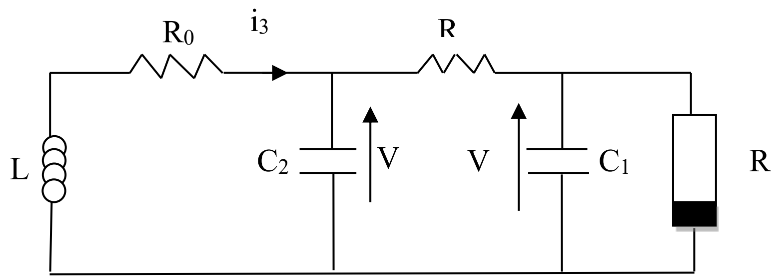

Figure 2. Chua Circuit

Chua system is represented by the set of differential equations:

$$\begin{cases} \dot{x} = \alpha(y - x - f(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -\beta(x - R_0 z) \end{cases} \quad (6)$$

$$\text{Where:} f(x) = m_1 x + \frac{m_0 - m_1}{2}(|x+1| - |x-1|) (7)$$

We will estimate the parameters α and β by the PSO optimization algorithm using the synchronization error between the transmitter and the receiver, because the synchronization error of chaotic systems is very sensitive to the parameters of the chaotic system. In this section we apply the synchronization technique theory of Pécora and Carroll between the two identical chaotic systems [19]. This synchronization is based on the "Master-Slave" concept. A Slave signal is intended to faithfully reproduce the Master signal. The "Master" system is also called transmitter and the "Slave" system is called receiver.

The receiver is a system identical to the transmitter in addition to be a simple sub-tractor in order to successfully remove the chaotic masking. This scheme is based on sending a driving signalX_m, which is a simple addition between the transmitter output signal y(t) and the message m(t). The signal s(t) is transmitted to the receiver through the transmission channel, assuming that only this channel is ideal, see Fig.3.

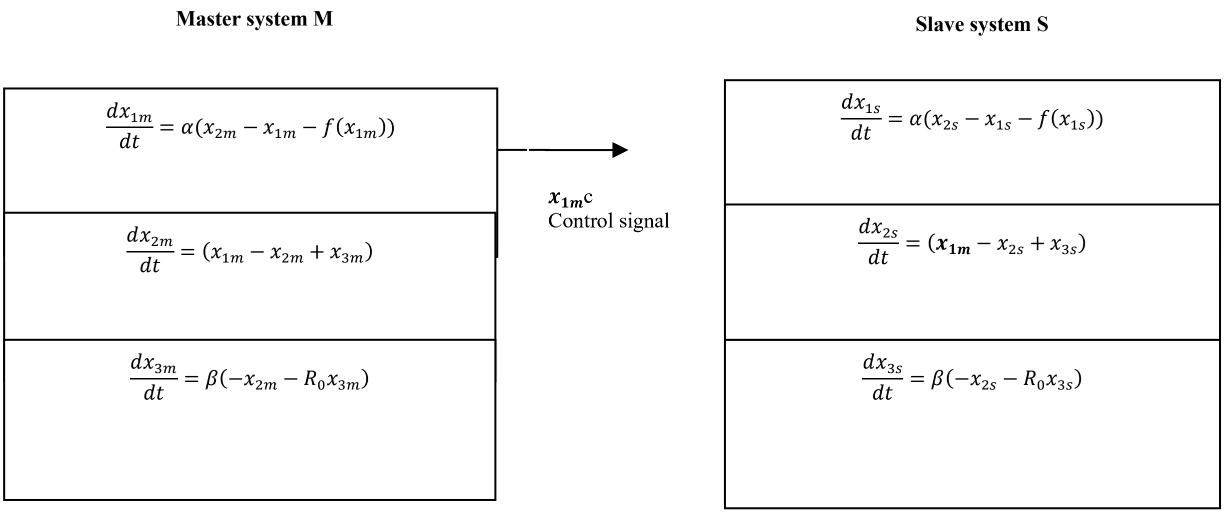

Figure 3. Pecora and Caroll synchronization method

Where:

x_m, y_m, z_m: master system state variables

x_s, y_s, z_s: slave system state variables

The synchronization is obtained when the error converges to zero in function with time tends to infinity, i.e.:

$$\lim_{t\to\infty} e(t) \qquad \text{where} \qquad e(t) = x_{s(k)}(t) - x_{m(k)}(t)$$

k is the state number

PSO Algorithms is used to optimize this objective function defined, as follows in (8), by the summation of the squared errors between master and slave systems [20-25]:

$$J = \frac{1}{M}\sum_{K=1}^{M} \|x_{sk} - x_{mk}\|^2 \qquad (8)$$

5. Simulation Results

In this part, the performances of the proposed method will be evaluated by experimental tests using a computer PC with Windows 7, 32 bits and the Matlab-13 software. We estimate the Chaotic Chua system parameters by the PSO optimization algorithm and we will check the efficiency of this method by a comparison with GA and GOW algorithms.

In order to study the various modes or the different behaviors that can be shown by the Chua circuit, it is enough to modify one of the Chua circuit parameters values, where we take the parameter α as a bifurcation parameter. We performed simulations by varying the parameter α from 0 to 19, with an iteration step of 0.01, $= 28 m_0 = -1.27$, $m_1 = -0.68$, and the initial conditions $(x_0, y_0 \text{ and } z_0) = (0.5, 1 \text{ and } -0.5)$. These presents different behaviours :

- ❖ For $\alpha = [0\ 13]$,we notice that the behavior of the Chua circuit converges towards a periodic solution. In this case, the trajectory converges towards a cycle of order 2, see Fig.4(a).
- ❖ By increasing values of $\alpha = [13.5\ 14]$, a new behavior converges to a periodic double solution with period doubling, see Fig.4(b).
- ❖ By increasing values of$\alpha = [14.3\ 14.9]$, a new behavior converges to a quasi-periodic solution with period doubling, see Fig.4(b)
- ❖ For $\alpha = [15\ 19]$,Chua's circuit behavior no longer represents an ordered structure. So, the system becomes chaotic as shown in Fig.4 (d).

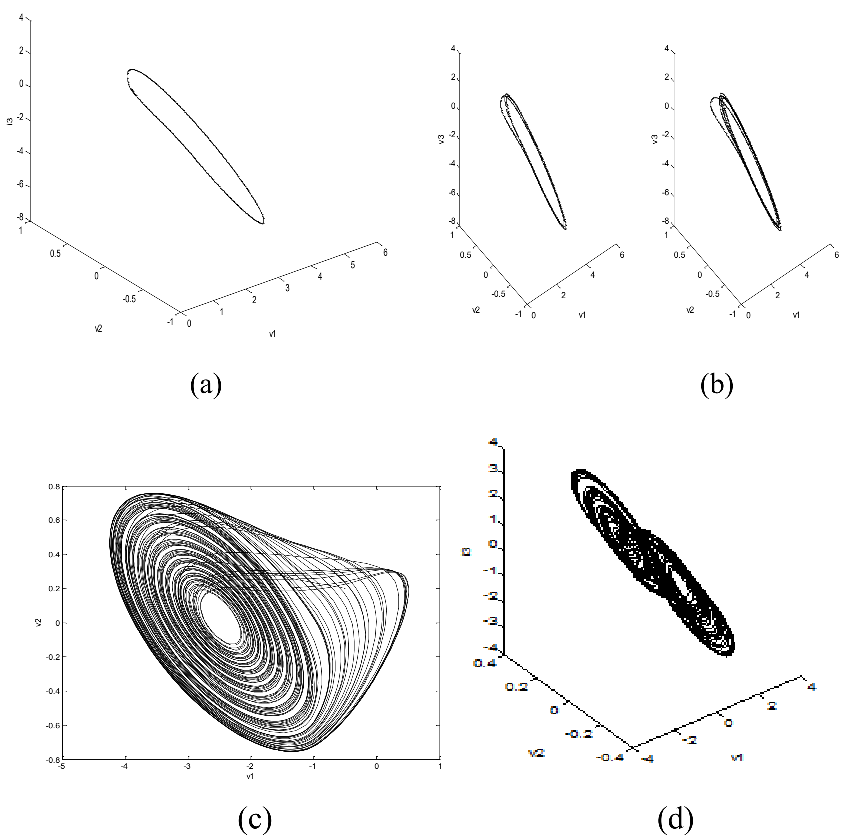

Figure 4. Chua circuit behaviour for different values of α

The different choices of PSO simulation parameters are given in Table.2, with time laps of 0.02 for 1000 steps. From the above, we will conclude that the Chua circuit parameters α and β which give the chaotic behavior of the Chua circuit lie in the approximate range of values 15≤α≤19 and 24≤β≤30.From this and in order to increase the dynamics of chaos, we will estimate these parameters by the PSO algorithm. We choose PSO simulation algorithm parameters given in Table.2, with a time step of 0.02 for 1000 steps.

PSO is used to estimate the parameters $\theta = (\alpha, \beta)^T$ in a range around the real values of the parameters in a chaotic regime, the search ranges of the parameters are defined as follows:

$$14 \leq \alpha \leq 17 \text{and } 26 \leq \beta \leq 30$$

- To estimate the parameter ∂, it is considered that the parameter β is known in advance with the original value. The initial assumptions for the control parameter α are in the range [14, 17].
- As before, and to estimate β parameter, we will only consider the variation on the parameter β and we assume α constant. Initial assumptions for the control parameter β are in the range [26,30].

The θ parameters were estimated using the PSO algorithm, where in each time we take three fine domains from the search range of α,β parameter values in a chaotic regime and we will make a comparison for the three versions of PSO.

We will run each algorithm 20 times and note the results obtained from the objective function J with the smallest population size. For each particle position (x) consists of m real numbers in the corresponding range, each individual of the population represents the possible solution of the objective function J minimization problem, which corresponds to the synchronization error of two master and slave systems in the chaotic mode as shown in Fig.3.

Tables 1 and 2 show the statistical values (best, maximum and minimum) of the estimated parameters α and β for each model of the PSO and GA algorithms.

Table 1. Statistical values of α parameter

Algorithm	α parameters		
	Best	Max	Min
PSO1/GA1	15.352	16.830	14.590
PSO2/GA2	15.800	16.530	14.550
PSO3/GA3	15.400	16.641	15.020

Table 2. Statistical values of β parameter

	β parameters		
Algorithm	Best	Max	Min
PSO1/GA1	27.820	29.513	26.950
PSO2/GA2	28.230	30.000	27.670
PSO3/GA3	27.950	28.130	26.653

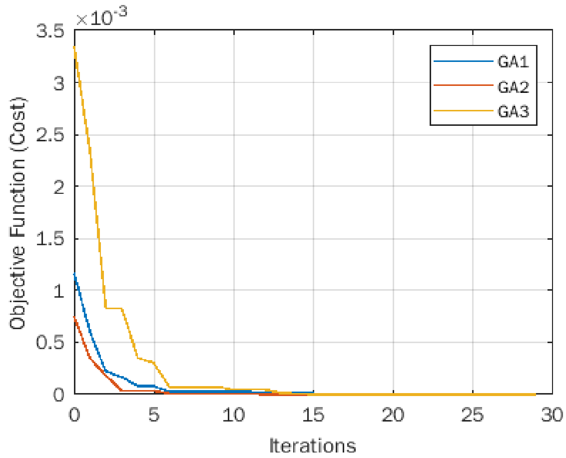

Figure 5. Objective function J evolution for α using GA (1-3) models

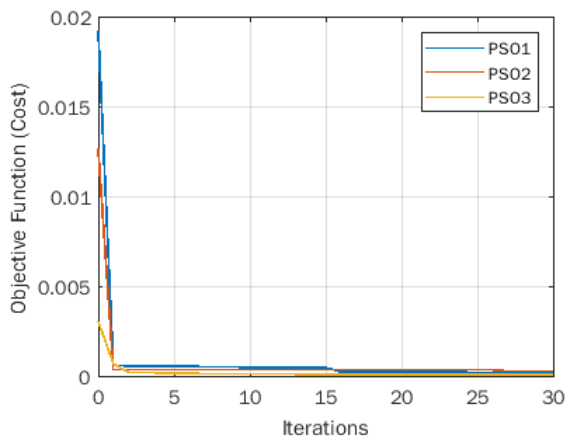

Figure 6. Objective function J evolution for α using PSO (1-3) models

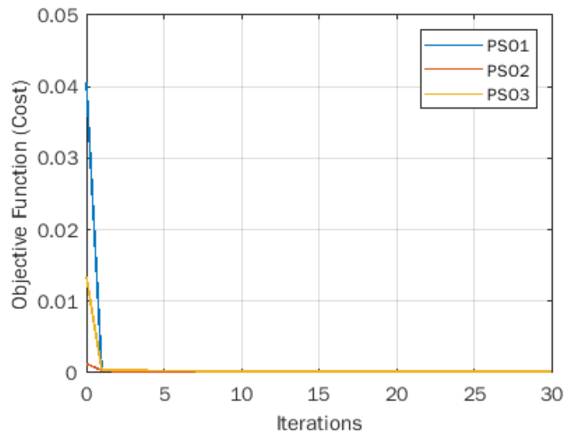

Figure 7. Objective function J evolution for β using PSO (1-3) models

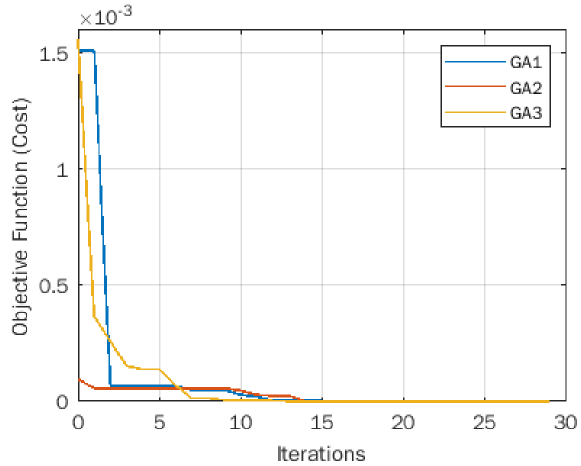

Figure 8. Objective function J evolution for β using GA (1-3) models

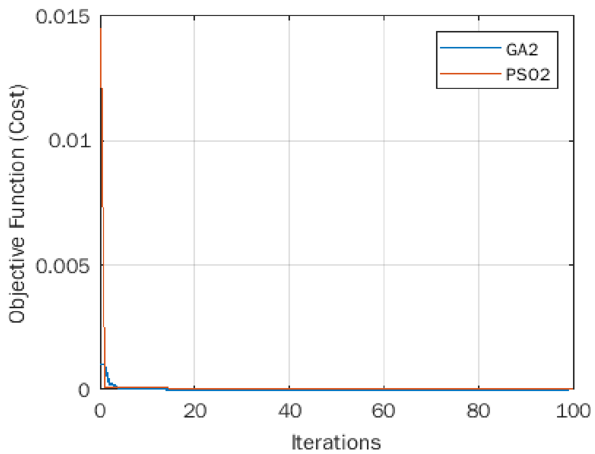

Figure 9. Objective function J evolution for $\alpha = 15.8, \beta = 28.23$

Figures: Fig.5, Fig.6, Fig.7, Fig.8 and Fig.9 show the objective function J evolution for the typical execution of the PSO(1-3) and GA(1-3) models with the values of the parameters α and β respectively.

We note that after few iterations, the best results (estimated values) are given by the PSO-2 model which offers the best values of the objective function quickly approaches the lowest value that tends to zero.

From the values estimated by PSO-2, α =15.8,β=28.23, we notice that the chaotic dynamics is increased compared to the chaotic dynamics of the Chua system with the real parameters as shown in Fig.10.

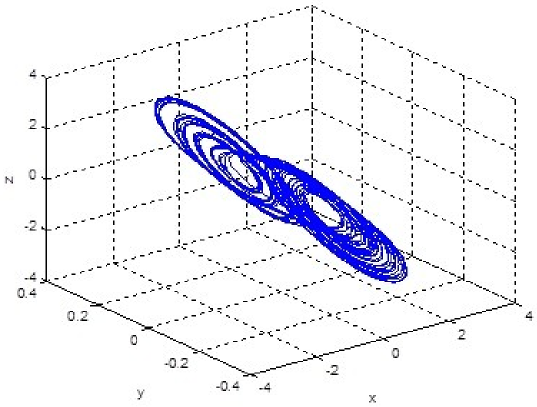

Figure 10. The new Chua chaotic system using the resultant estimated: $\alpha = 15.8, \beta = 28.23$

6. Conclusion

In this paper we have proposed and applied a PSO optimization algorithm to estimate the parameters of a chaotic Chua system. We considered the synchronization error between the master system and the slave system as an optimization problem, the performance of the PSO algorithm is compared to GA. The results of the simulation demonstrated that the proposed algorithm performs better than others in estimating the chaotic system parameters of Chua which increase the dynamics of chaos, in order to increase the security of communication systems in the event of chaos.

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